Consumers Loans and Math 101 Course Content

Consider the consumer loan process. It's very simple. You need some money, so you go to some institution that's willing to lend it to you. They give you a big chunk of money up front, and you have to pay it back at some fixed amount each month. Every month, the lender calculates how much interest you owe for that month, by multiplying your "balance" times the agreed-upon interest rate. The interest gets subtracted from your monthly payment. Whatever is left over goes toward paying back the loan, thus reducing the balance for the next month. This process continues until the balance drops to zero, at which point the loan is paid off.

Obviously there is some relationship between the loan amount, the interest rate, the number of monthly payments, and the monthly payment amount. Given any three of those numbers, the fourth number is completely determined by the process – there's only one value that will make the loan be exactly paid off after the specified number of payments.

So, there are at least four obvious questions:

(A) How much would this cost me? (Find the monthly payment amount, given the loan amount, interest rate, and number of payments.)

(B) How much can I borrow? (Find the loan amount, given the interest rate, number of payments, and monthly payment amount.)

(C) How long am I stuck for? (Find the number of payments, given the loan amount, interest rate, and monthly payment amount.)

(D) How good of a deal do I need? (Find the interest rate, given the loan amount, number of payments, and monthly payment amount.)

It turns out that quite different methods are required to answer these four questions.

For question (A) – find the monthly payment – there's a "simple" formula. The formula is surprisingly difficult to derive. (Your instructor has never succeeded!) But it's not difficult to understand, and if you were desperate, the formula could be evaluated by hand or with a basic desk calculator in a few minutes.

$$P = \frac{L \cdot r \cdot (1+r)^N}{(1+r)^N - 1}$$

(for P = monthly payment, L = loan amount, r = monthly interest rate, N = number of payments)

Given this formula, it's easy to answer question (B) — find the loan amount — simply by rearranging the equation. (This is classic algebra.)

$$L = \frac{P \cdot \left((1+r)^N - 1 \right)}{r \cdot (1+r)^N}$$

For question (C) — find the number of payments — there is also a formula, but it's rather more cryptic and involves the "logarithm" function:

$$N = \frac{\ln\left(\frac{P}{P - Lr} \right)}{\ln(1+r)}$$

And finally, for question (D) — find the interest rate — there simply is no formula to do it! The interest rate can be computed without much difficulty, but it requires a numerical approach called "successive approximation" — a particularly well organized form of "guess and test".

This particular topic — consumer loans — is of such wide interest that tools can easily be found to do the required calculations for you. Unfortunately, that is not true for many other problems that crop up in both careers and in daily living.

So, as an example of an EXTREMELY powerful technique, we will study how all four numbers in the consumer loan problem can be determined quite easily by using simple, straightforward calculations in a spreadsheet to essentially simulate the entire loan process, combined with successive approximation to find numeric solutions.

The same strategy — spreadsheet calculations combined with automatic "solvers" built into the spreadsheet software — works very well to address a wide variety of other problems for which prepackaged tools do not exist or cannot be found.

This technique is not discussed in educational textbooks. 15 years ago, it was easy to explain this lack by asserting that spreadsheets and solvers were too specialized, or too uncommon, or that they somehow got in the way of understanding, or that they were just the "wave of the future" and it was premature to talk about them.

Today, spreadsheets and solvers are widely available and easily used, but they are sadly underutilized. It's time to fix that. They will play a major role in this course.