Excel for Algebra [1]

Lesson 3: Consumer Loan Simulation

What is this lesson about?

On the first day of class, we looked very briefly at the consumer loan process as an example of interesting mathematics.

I showed you a spreadsheet that simulated the process of paying back a loan, and I used Goal Seek on that spreadsheet to answer several different questions.

In today's lesson, we will re-create that spreadsheet from scratch.

This will serve two purposes:

1. Give you some practice constructing tables in Excel.
2. Provide some interesting and practical examples of using Goal Seek

A brief refresher: how consumer loans work

Consumer loans are simple in concept. You need some money, so you go to an institution that's willing to lend it to you. They give you a big chunk of money up front, and you have to pay it back at some fixed amount each month. Every month, the lender figures out how much interest you owe for that month, by multiplying your "balance" times the agreed-upon interest rate. The interest gets subtracted from your monthly payment. Whatever is left over goes toward paying back the loan, thus reducing the balance for the next month. This process continues until the balance drops to zero, at which point the loan is paid off.

Loan computations

Four numbers are involved in the basic consumer loan process:

- The loan amount.
- The interest rate, typically expressed as % per year.
- Your monthly payment.
- The number of months until the loan is paid back.

These four numbers are closely related, of course — given any three of them, the fourth number can be determined by one means or another. Two of the four numbers (loan amount and monthly payment) can be calculated with rather simple formulas involving just arithmetic. The third one

(number of months) can be calculated by a formula involving the logarithm function. For the fourth number (interest rate), there simply is no formula — calculating interest rate from the other three numbers can only be done by successive approximation, using Goal Seek or something similar.[2]

Building a spreadsheet to simulate paying back the loan

Simulation turns out to be a wonderful way to attack lots of problems. It's very flexible and easy to understand — which also means that it's reliable and easy to check.

Here is an example of the spreadsheet that we're going to build:

	A	B	C	D	E
1	Consumer Loan Simulation				
2					
3	Loan Amount	1000			
4	Annual Interest Rate	0.18			
5	Monthly Interest Rate	0.015			
6	Monthly Payment	91.68			
7					
8	Month Number	Monthly Starting Balance	Monthly Interest	Payment Toward Principal	Monthly Ending Balance
9	1	1000.00	15.00	76.68	923.32
10	2	923.32	13.85	77.83	845.49
11	3	845.49	12.68	79.00	766.49
12	4	766.49	11.50	80.18	686.31
13	5	686.31	10.29	81.39	604.92
14	6	604.92	9.07	82.61	522.32
15	7	522.32	7.83	83.85	438.47
16	8	438.47	6.58	85.10	353.37
17	9	353.37	5.30	86.38	266.99
18	10	266.99	4.00	87.68	179.32
19	11	179.32	2.69	88.99	90.33
20	12	90.33	1.35	90.33	0.00

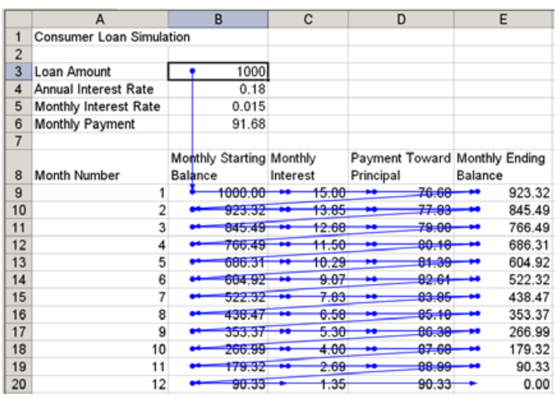
As you can see, this spreadsheet simulates the paying back of a $1000 loan, at 18% annual interest rate. A monthly payment of $91.68 exactly pays off the loan at the end of 12 months.

[2] Financial calculators have a built-in function to calculate interest rate given loan amount, monthly payment, and number of months. Inside the calculator, that function is computed using successive approximation.

Here are the formulas in the spreadsheet.

	A	B	C	D	E
1	Consumer Loan Simula				
2					
3	Loan Amount	1000			
4	Annual Interest Rate	0.18			
5	Monthly Interest Rate	=B4/12			
6	Monthly Payment	91.68			
7					
8	Month Number	Monthly Starting Balance	Monthly Interest	Payment Toward Principal	Monthly Ending Balance
9	1	=B3	=B9*$B$5	=$B$6-C9	=B9-D9
10	2	=E9	=B10*$B$5	=$B$6-C10	=B10-D10
11	3	=E10	=B11*$B$5	=$B$6-C11	=B11-D11
12	4	=E11	=B12*$B$5	=$B$6-C12	=B12-D12
13	5	=E12	=B13*$B$5	=$B$6-C13	=B13-D13
14	6	=E13	=B14*$B$5	=$B$6-C14	=B14-D14
15	7	=E14	=B15*$B$5	=$B$6-C15	=B15-D15
16	8	=E15	=B16*$B$5	=$B$6-C16	=B16-D16
17	9	=E16	=B17*$B$5	=$B$6-C17	=B17-D17
18	10	=E17	=B18*$B$5	=$B$6-C18	=B18-D18
19	11	=E18	=B19*$B$5	=$B$6-C19	=B19-D19
20	12	=E19	=B20*$B$5	=$B$6-C20	=B20-D20

These arrows will probably help visualize how the data flows through the spreadsheet:



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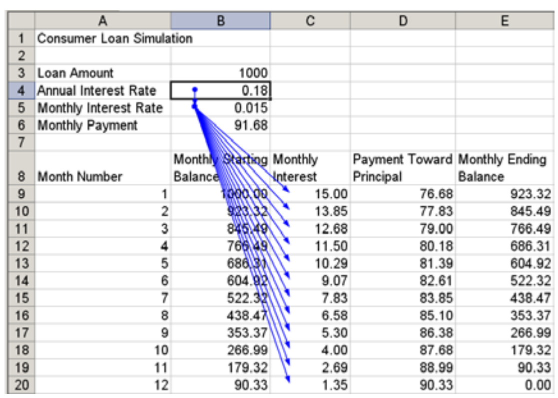

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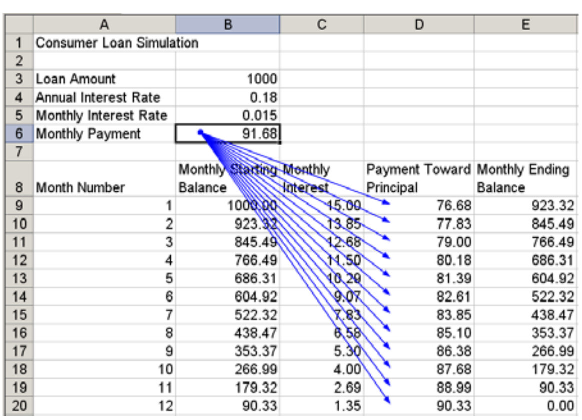

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To construct this spreadsheet, proceed as follows...

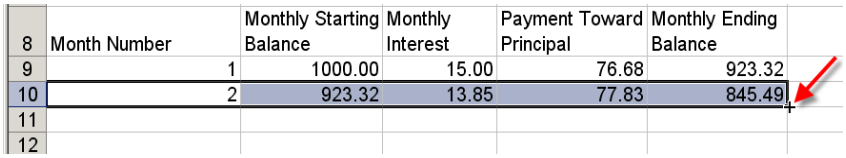
First, select cells B9 through E20, and format them with Format > Cells > Number, with 2 decimal places. This will avoid lots of confusing extra digits as we go through this.

Now, rows 1 through 8 are easily constructed by just typing. There's only one formula in these rows, in cell B5 (=B4/12, turning annual interest rate into monthly interest rate).

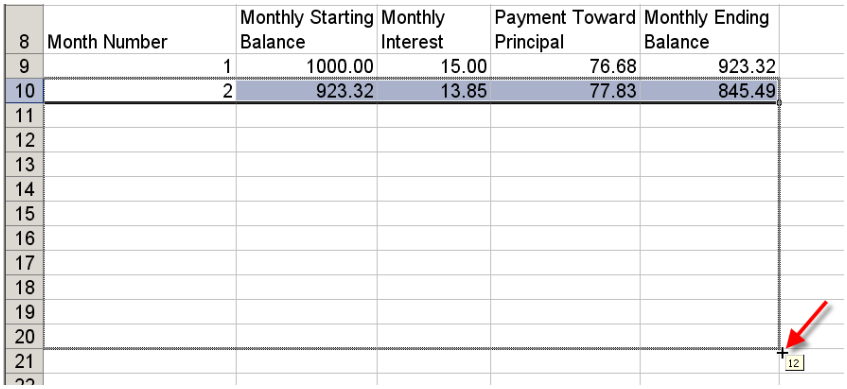
Rows 9 and 10 can be constructed either by typing or clicking on cells, as you prefer.

IMPORTANT: the references to B5 and B6 (interest rate and monthly payment) are written with dollar signs: **$B$5** and **$B$6**. Those dollar signs make the cell references absolute, which means that they continue to point to B5 and B6 even when the formulas are moved, copied, or stretched to occupy other cells in the spreadsheet. If you construct the formulas by typing, just type in the dollar signs. If you construct the formulas by clicking, you can add the dollar signs either by hitting the F4 function key, or by typing the dollar signs later — typically after you realize that you accidentally left them out.

To construct rows 11-20, simply select cells A10:E10, position your cursor over the little box in the lower right corner, press, and stretch the formulas downward. As you stretch the formulas, it looks like this. (The little "12" in the box shows the value that would appear in cell A20, at lower left corner, when you release the mouse.)



8	Month Number	Monthly Starting Balance	Monthly Interest	Payment Toward Principal	Monthly Ending Balance
9	1	1000.00	15.00	76.68	923.32
10	2	923.32	13.85	77.83	845.49
11					
12					



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10	2	923.32	13.85	77.83	845.49
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12					
13					
14					
15					
16					
17					
18					
19					
20					
21					

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16	8	438.47	6.58	85.10	353.37
17	9	353.37	5.30	86.38	266.99
18	10	266.99	4.00	87.68	179.32
19	11	179.32	2.69	88.99	90.33
20	12	90.33	1.35	90.33	0.00

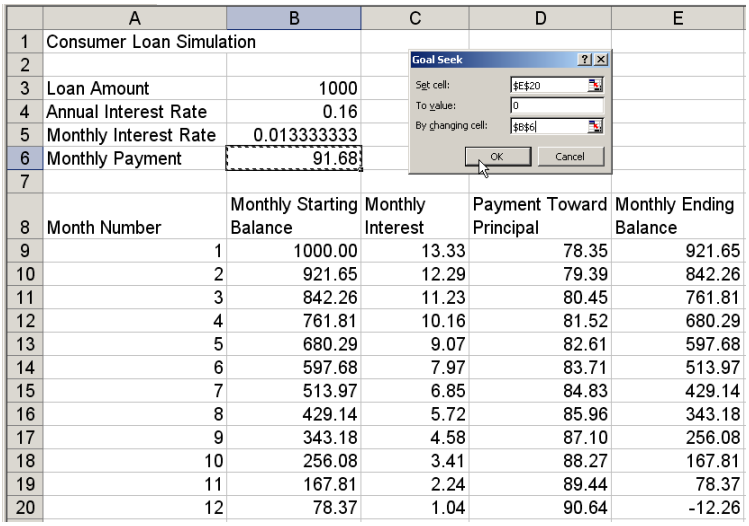
At this point, you're done constructing the spreadsheet. **Save it now** – just in case you mess it up too much later.

Exploring alternatives with Goal Seek

It's easy to use this spreadsheet to explore alternatives, using Goal Seek.

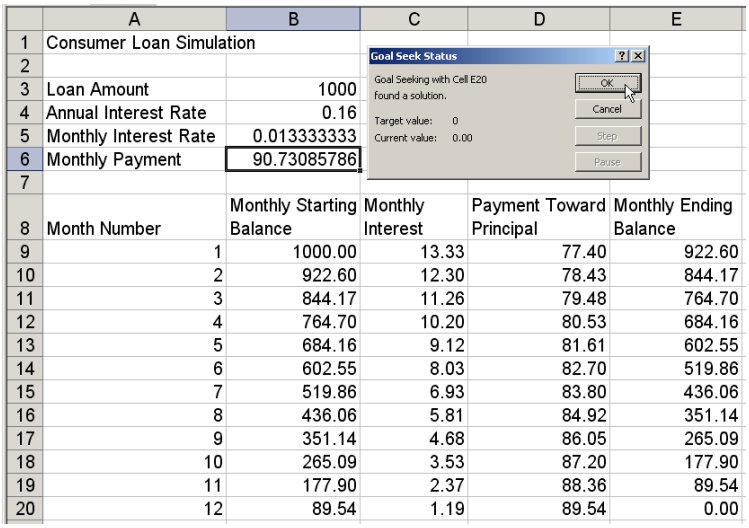
The basic rule is that you want the loan exactly paid off at the end of the term. For a 12-month loan, that means we want cell E20 = 0, that is, Monthly Ending Balance equal zero, at Month Number 12.

Let's change the interest rate to 16%, and see what the monthly payment would have to be. Before we let Goal Seek do its job, we see that the old monthly payment of $91.68 was a little too big — the loan was more than paid off at the end of 12 months.



	A	B	C	D	E
1	Consumer Loan Simulation				
2					
3	Loan Amount	1000			
4	Annual Interest Rate	0.16			
5	Monthly Interest Rate	0.013333333			
6	Monthly Payment	91.68			
7					
8	Month Number	Monthly Starting Balance	Monthly Interest	Payment Toward Principal	Monthly Ending Balance
9	1	1000.00	13.33	78.35	921.65
10	2	921.65	12.29	79.39	842.26
11	3	842.26	11.23	80.45	761.81
12	4	761.81	10.16	81.52	680.29
13	5	680.29	9.07	82.61	597.68
14	6	597.68	7.97	83.71	513.97
15	7	513.97	6.85	84.83	429.14
16	8	429.14	5.72	85.96	343.18
17	9	343.18	4.58	87.10	256.08
18	10	256.08	3.41	88.27	167.81
19	11	167.81	2.24	89.44	78.37
20	12	78.37	1.04	90.64	-12.26

After we Goal Seek, it's a different story. The new monthly payment is $90.73.



	A	B	C	D	E
1	Consumer Loan Simulation				
2					
3	Loan Amount	1000			
4	Annual Interest Rate	0.16			
5	Monthly Interest Rate	0.013333333			
6	Monthly Payment	90.73085786			
7					
8	Month Number	Monthly Starting Balance	Monthly Interest	Payment Toward Principal	Monthly Ending Balance
9	1	1000.00	13.33	77.40	922.60
10	2	922.60	12.30	78.43	844.17
11	3	844.17	11.26	79.48	764.70
12	4	764.70	10.20	80.53	684.16
13	5	684.16	9.12	81.61	602.55
14	6	602.55	8.03	82.70	519.86
15	7	519.86	6.93	83.80	436.06
16	8	436.06	5.81	84.92	351.14
17	9	351.14	4.68	86.05	265.09
18	10	265.09	3.53	87.20	177.90
19	11	177.90	2.37	88.36	89.54
20	12	89.54	1.19	89.54	0.00

The same technique can be used to find values for the loan amount and interest rate.

To find the number of months, simply stretch the formulas across as many rows as you need, and look for where the Monthly Ending Balance either goes to 0, or changes sign from + to – .

Caution: When you do the stretch, select only the **last** line of the table, and stretch that down. If you select the entire table by mistake, then the special-case formulas in the first line get replicated farther down in the table, and things get seriously messed up.