Excel for Algebra [1]

Lesson 2: Checking Algebra By Plugging In Numbers

Objective

Learn how to use Excel to check symbolic algebra by plugging in numbers.

Discussion

Most of algebra is concerned with changing one arrangement of symbols into another arrangement that looks different but is somehow "equivalent". There are two common cases:

Case 1: Rewriting an <u>expression</u> in some way that does not change its numeric value. For example *a*(b+c)* is equivalent to *a*b+a*c* because those two expressions always end up producing the same value no matter what values are used for *a*, *b*, and *c*. This case includes operations like "factoring", "distributing", "combining like terms", and "cancelling common factors".

Case 2: Rewriting an <u>equation</u> in some way that may change the numeric values on the left and right sides, but preserves the variable values that make the equations true. For example, $a = b + c$ is equivalent to $a - c = b$ because whenever *a*, *b*, and *c* have values that make the first equation true, then the second equation is true also, and vice versa. This case covers the final result of "solving an equation for a specified variable", as well as other steps of that process where you "do the same thing to both sides".

Unfortunately, people's brains are not good at doing this sort of work. Mistakes happen frequently. As a result, checking the work is critical! But how?

Basic arithmetic provides a check that is both simple and powerful.

Here's the ritual:

Case 1: (two expressions) Choose a different random value for each variable. Do the arithmetic for each expression. Check that both expressions produce the same value.

Case 2: (two equations) Choose random values for every variable except one. Do the arithmetic for the first equation and determine a value for that last variable so that the first equation becomes true. Then plug those same values into the second equation, do the arithmetic, and check that the second equation is true also, for those very same values of the variables.

We're going to need Case 2 a lot more often, so that's what this lesson will concentrate on.

Example #1: Confirming a correct answer

Suppose we have this problem: "Given the equation $F = \frac{9}{5}C + 32$, solve for *C*."

After rearranging the symbols, we decide that the answer is probably $C = \frac{5}{9}(F - 32)$.
Is that answer correct?

To check, we plug in numbers.

First, choose any convenient values for *C*, say *C*=15. Plug that value into the first equation $F = \frac{9}{5}C + 32$, do the arithmetic, and find that $F = \frac{9}{5} \cdot 15 + 32 = 27 + 32 = 59$. Then plug those same values into the second equation and do the arithmetic to see whether $15 = \frac{5}{9}(59 - 32)$. It does, so the answer $C = \frac{5}{9}(F - 32)$ checks OK.

To make the check more iron-clad, we can repeat the process with different values. Say *C=29*. Then plugging into the first equation gives $F = \frac{9}{5} \cdot 29 + 32 = 84.2$. The check is to see whether the second equation works too. Does $29 = \frac{5}{9}(84.2 - 32)$? Doing the arithmetic, we find that it does, so again the equation checks OK. It also checks OK for any other values we choose.

Because the same numbers work in both equations, we can now be confident about the algebra. Yes, the answer is correct — we checked it.

If the algebra were wrong, then the numbers almost certainly would not work. For example if we made a sign error and erroneously concluded that $C = \frac{5}{9}(F + 32)$, then we would try to confirm that $15 = \frac{5}{9}(59 + 32)$, and it simply doesn't. The fact that the numbers do not work with that formula tells us that the something is wrong.

Well, this is pretty cool, using arithmetic to check algebra, but there are still a couple of problems. First, it's incredibly boring and tedious. Second (and more serious), if you do the arithmetic by hand or with a simple calculator, then it's very likely that you will mess up the arithmetic — quite possibly in the very same way you messed up the algebra — which again will invalidate the check.

A much better approach is to use a spreadsheet to do the arithmetic.

This will, of course, require you to correctly convert between visual notation and single-line notation. Be sure to include required parentheses when doing this conversion!

Here is a standard layout that we'll be using to check our solutions.

Values:

	A	B	C	D	E	F	G	H	I	J
1	**Example #1.**									
2										
3	**Confirming a correct solution**									
4										
5	We were given that F = (9/5)*C + 32 .									
6	Is it correct that C = (5/9)*(F - 32) ?									
7										
8	Variables	F	C							
9	Values	84.2	29							
10										
11	Equation	Left Side	Right Side	Difference						
12	F = (9/5)*C + 32 ?	84.2	84.2	0	<---- zero indicates that left side = right side, the equation is true					
13	C = (5/9)*(F - 32) ?	29	29	0	<---- zero indicates that left side = right side, the equation is true					
14										
15	I tested that both equations are true for all these values also.									
16		F	C							
17		59	15							
18		23	-5							
19		-22	-30							

Formulas:

	A	B	C	D	E	F	G
1	**Example #1.**						
2							
3	**Confirming a correct solution**						
4							
5	We were given that F = (9/5)*C + 32 .						
6	Is it correct that C = (5/9)*(F - 32) ?						
7							
8	Variables	F	C				
9	Values	84.2	29				
10							
11	Equation	Left Side	Right Side	Difference			
12	F = (9/5)*C + 32 ?	=B9	=(9/5)*C9+32	=B12-C12	<---- zero indicates that left side = right side, the equation is true		
13	C = (5/9)*(F - 32) ?	=C9	=(5/9)*(B9-32)	=B13-C13	<---- zero indicates that left side = right side, the equation is true		
14							
15	I tested that both equations are true for all these values also.						
16		F	C				
17		59	15				
18		23	-5				
19		-22	-30				

Take careful note of the following points, to demonstrate that an answer is correct:

- **<u>Both</u> equations are tested using the <u>same</u> values of all variables.**
- **Both equations are <u>true</u> for those values.**

Exercise #1. Reconstruct the above spreadsheet, starting from scratch. Be sure to pick some values of your own for F and C. Don't just use the ones that are shown.

Exercise #2. Construct another spreadsheet, with the same structure, to check that the following equation has been solved correctly:

Given: $A = L\,W$

Is it correct? $W = A / L$

Detecting and tracking down a mistake

Now suppose that we messed up our algebra, left out some parentheses in our algebra, and ended up with the following incorrect result:

(Caution: There are deliberate mistakes in this table!)

Label	**Equation**	**Explanation of where the equation comes from**
Eq #1	$F = \frac{9}{5}C + 32$	given
Eq #2	$F - 32 = \frac{9}{5}C$	subtract 32 from both sides
Eq #3	$\frac{5}{9} \cdot F - 32 = \frac{5}{9} \cdot \frac{9}{5}C$	multiply both sides by the inverse of 9/5. *(Required parentheses have been left out here!)*
Eq #4	$\frac{5}{9} \cdot F - 32 = 1 \cdot C$	simplify the right hand side using associative rule for multiplication and also the rule that product of inverses = 1
Eq #5	$C = \frac{5}{9}F - 32$	swap sides, noting that $1 \cdot C = C$ because 1 is the multiplicative identity

Here is what happens when we check the <u>incorrect</u> result. Notice that line 35 is true but line 36 is false. **Having one equation true but the other false indicates a mistake.**

Values:

	A	B	C	D	E
21	**Detecting a mistake**				
22					
23	We were given that F = (9/5)*C + 32 .				
24	Is it correct that C = (5/9)*F - 32 ? (Warning! Some parentheses are missing here.)				
25					
26	Pluck a value for C out of the air, and compute a matching value of F using the given formula.				
27					
28	Variables	F	C		
29	Values	59	15		
30					
31	Now check to see whether the equations are satisfied.				
32					
33	Equation	Left Side	Right Side	Difference	
34					
35	F = (9/5)*C + 32 ?	59	59	0	<---- zero indicates that left side = right side, the equation is true
36	C = (5/9)*F - 32 ?	15	0.777778	14.22222	<---- not zero means that left side <> right side, the equation is false!
37					
38	**Tracking down where things went wrong**				
39					
40	Let's check "Eq # 3"				
41		Left Side	Right Side	Difference	
42	(5/9)*F-32 = (5/9)*(9/5)*C ?	0.77778	15	-14.22222	<---- not zero: rats, that one is wrong too!
43					
44	OK, how about "Eq # 2" ?				
45		Left Side	Right Side	Difference	
46	F-32 = (9/5)*C ?	27	27	0	<---- zero difference: OK, that one checks.
47					
48	Eq # 2 also checks for any other matching values of C and F that we plug in.				
49	So apparently the error was made between Eq #2 and Eq #3. Now it's pretty easy to find.				

Formulas:

	A	B	C	D	E	F	G
21	**Detecting a mistake**						
22							
23	We were given that F = (9/5)*C + 32 .						
24	Is it correct that C = (5/9)*F - 32 ? (Warning! Some parentheses are missing here.)						
25							
26	Pluck a value for C out of the air, and compute a matching value of F using the given formula.						
27							
28	Variables	F	C				
29	Values	59	15				
30							
31	Now check to see whether the equations are satisfied.						
32							
33	Equation	Left Side	Right Side	Difference			
34							
35	F = (9/5)*C + 32 ?	=B29	=(9/5)*C29+32	=B35-C35	<---- zero indicates that left side = right side, the equation is true		
36	C = (5/9)*F - 32 ?	=C29	=(5/9)*B29 - 32	=B36-C36	<---- not zero means that left side <> right side, the equation is false!		
37							
38	**Tracking down where things went wrong**						
39							
40	Let's check "Eq #3"						
41		Left Side	Right Side	Difference			
42	(5/9)*F-32 = (5/9)*(9/5)*C ?	=(5/9)*B29-32	=(5/9)*(9/5)*C29	=B42-C42	<---- not zero: rats, that one is wrong too!		
43							
44	OK, how about "Eq #2" ?						
45		Left Side	Right Side	Difference			
46	F-32 = (9/5)*C ?	=B29-32	=(9/5)*C29	=B46-C46	<---- zero difference: OK, that one checks.		

Once a mistake has been detected, the checking process can be used to narrow down where it was made. What is shown in the spreadsheet is to determine that Eq #3 is also incorrect, but Eq #2 is OK. That means the mistake occurred in going from Eq #3 to Eq #2, and once we know that, it's pretty easy to find — that's where we left out required parentheses.

Re-working the algebra using the corrected equation gives us this derivation:

Equation	**Explanation of where the equation comes from**
$F = \frac{9}{5}C + 32$	given
$F - 32 = \frac{9}{5}C$	subtract 32 from both sides
$\frac{5}{9}(F - 32) = \frac{5}{9}\left(\frac{9}{5}C\right)$	multiply both sides by the inverse of 9/5. *(Now the required parentheses have been included.)*
$\frac{5}{9}(F - 32) = 1 \cdot C$	simplify the right hand side using associative rule for multiplication and the rule that product of inverses = 1
$C = \frac{5}{9}(F - 32)$	swap sides, noting that $1 \cdot C = C$ because 1 is the multiplicative identity

This final result is the one that we checked a couple of pages back, using spreadsheet arithmetic to confirm that the symbolic algebra is correct.

Ways to screw up the check

To "screw up the check" means to construct a spreadsheet that superficially appears to be a check, but in fact does nothing useful. This is a grievous offense. There are several common ways to commit it:

- Test only one equation. (Both equations must be tested.)
- Test both equations, but use values that make both equations false. (At least one equation must be true. If both equations are false, then the test provides no useful information.)
- Test both equations, but use different values for each one. (Both equations must be tested with the same values.)
- Use the Solver to find special combinations of values that make both equations true even though one or both equations are wrong. (We will study Solver later. For now, the important thing to know is never use Solver when checking equations. Solver is a tool for solving equations, not for checking them.)

Finding numbers to use for the check

To do a valid check, we need to find numbers that make one of the equations true.

There are two good ways to find such numbers:

- In most problems, one equation has some variable isolated on the left. In that case, you can just pick arbitrary values for the other variables, plug them in, and do the arithmetic to find out what value the isolated variable must have. Once you know the correct number, you can either:

  a) Type that number into the appropriate "Values" cell, or

  b) Use Excel commands "Copy" and "Paste Special...Values" to copy the number from wherever it is calculated up into the appropriate "Values" cell. ("Paste Special..." appears in a popup menu when you right-click on a cell.)

  Paste Special...Values is the best approach because it guarantees that all available digits will be copied, instead of just the ones you can see.

  But be careful: if you use ordinary Paste instead of Paste Special...Values, the result will be chaos because what gets pasted will be an altered formula, with all its cell references messed up!

- In some problems, neither equation will have an isolated variable. In that case, you can pick arbitrary values for all variables except one, plug them in, then use Excel's "Goal Seek" command to find the required value for the last variable. This method is very powerful, but it is also more difficult to use because it produces answers that are less accurate than direct calculation.

There are also some general rules for choosing good numbers to use for tests:

- Avoid the numbers 0, 1, and any number that appears in either equation.
- Do not use the same number for two different variables at the same time.
- If possible, be sure to test with at least two different values for each variable.

Dealing with approximate arithmetic

It's an unfortunate fact of life that numbers in spreadsheets often are not exact.

This is particularly common if you copy values by retyping them, because you probably won't type all 15 digits that Excel uses internally.

But even when you do not retype any numbers, sometimes rounding within the computer will produce two results that are slightly different even though logically (using exact numbers) they would be the same. An example in Excel is *A*(B+(C+D))* versus *A*((B+C)+D)*, where *A*=1, *B*=-0.5, *C*=0.4, and *D*=0.1. Using exact arithmetic, these expressions produce the same answer, zero. But in Excel, only the first expression produces zero. The second expression produces 2.77556E-17. The difference is very small, 0.0000000000000000277556, but if you were expecting it to be exactly zero you will be surprised and may think something is wrong in the algebra.

One good approach is to consider how big the difference is when compared to the size of the biggest number going into the equations. If the difference is smaller than one millionth of the biggest number, then it's not worth worrying about. For example if your biggest number is 1000, then differences smaller than 0.001 are probably OK. If the difference is bigger than that, you should start worrying, and if it ever gets as big as one thousandth of the biggest number, then you almost certainly have a mistake.

Introducing Goal Seek:
What to do when you don't have an isolated variable

Recall that our basic scheme is to find numbers that make one equation true, then plug those same numbers into other equations and see if those equations are true also.

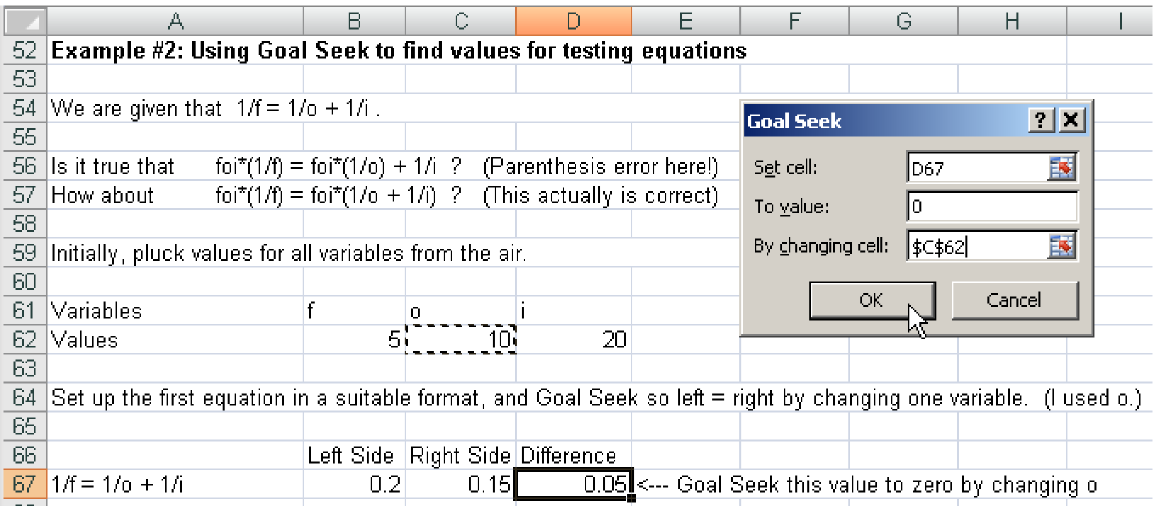
When one equation is a formula, something like $A = bh$ with one variable isolated by itself, then life is easy. Just do what we did up above: plug in values for all the other variables, do the arithmetic, and that's the value that the isolated variable must have.

But when the equations are <u>not</u> formulas, say they're something like $1/f = 1/o + 1/i$, then life gets a little harder. Fortunately, Excel (and most other spreadsheet programs) include a "Goal Seek" capability for finding numbers that solve problems. Using this capability, a good strategy is to pick values for all variables except one, then Goal Seek the remaining variable to make one equation true.

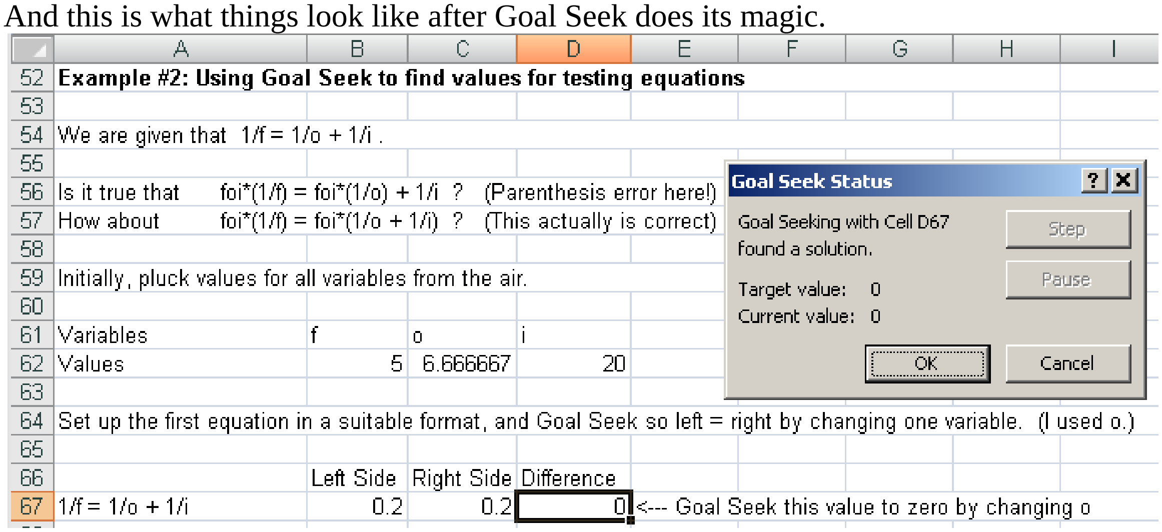
It turns out that you cannot directly tell Goal Seek to solve an equation. You can only tell Goal Seek to make one specified cell be some particular value. You cannot tell it to make two specified cells be the same unknown value. Fortunately, there is a simple way to work around this deficiency.

Just solve for Difference = 0. Since Difference = Left Side – Right Side, solving for Difference = 0 is the same as solving for Left Side = Right Side.

This is what things look like after we have set up the problem, but just before we have actually clicked OK to do the Goal Seek.



And this is what things look like after Goal Seek does its magic.



Well, that's the concept. Unfortunately, things don't usually work out quite so neatly.

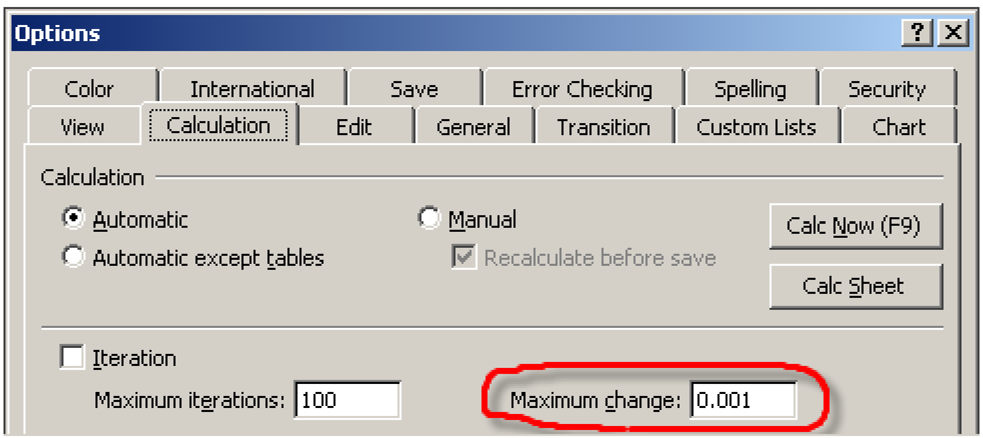
Most of the time, Goal Seek cannot push the Difference clear down to exactly zero because it's only working with 15-digit numbers. Worse, Excel's default settings are really pretty sloppy — it only tries to get the first 3 digits right!

So, when we try to solve this problem using Excel's default settings, what we get is something like this. (The exact values depend on which version of Excel you're using.)

61	Variables	f	o	i	
62	Values	5	6.669403	20	
63					
64	Set up the first equation in a suitable format, and Goal Seek so left = right by changing one variable. (I used o.)				
65					
66		Left Side	Right Side	Difference	
67	1/f = 1/o + 1/i	0.2	0.199938	6.153E-05	<--- Goal Seek this value to zero by changing o

Notice that Excel has gotten only the first three digits of o correct: 6.6694 versus 6.6666. As a result, the left side and right side are not exactly equal. They are different by a very small amount: 6.1521E-05, which actually means .000061521.

We can make Excel find a closer value by just telling it that we want one. To do that, go into Tools > Options > Calculation tab[2] and find the "Maximum change:" field.



Go to that "Maximum change" field and insert another 8 zeroes right after the decimal point. That will give you the number 0.00000000001 . (You won't be able to see all the digits at once. If you want to be sure what's there, scroll across using the arrow keys.)

With this smaller "Maximum change", Excel's Goal Seek will now find a value of *o* that is much more accurate.

Depending on the type of equation and on the starting guess, Goal Seek still may not be able to make the Difference be exactly zero. For example, starting with *o* = 10 now produces a difference of 1.38778E-15 = 0.00000000000000013877 . But this is a very small value indeed, even though it might not look like it at first glance. It is about as small as the diameter of a proton, compared with the width of this printed page!

[2] Tools > Options > Calculation is for Excel 2003. In Excel 2007, the Maximum change field is located at Office Button > Excel Options > Formulas.

Using this not-quite-perfect value for *o*, we get results that look like this:

	A	B	C	D	E	F	G	H	I	J
52	**Example #2: Using Goal Seek to find values for testing equations**									
53										
54	We are given that 1/f = 1/o + 1/i .									
55										
56	Is it true that foi*(1/f) = foi*(1/o) + 1/i ? (Parenthesis error here!)									
57	How about foi*(1/f) = foi*(1/o + 1/i) ? (This actually is correct)									
58										
59	Initially, pluck values for all variables from the air.									
60										
61	Variables	f	o	i						
62	Values	5	6.666667	20						
63										
64	Set up the first equation in a suitable format, and Goal Seek so left = right by changing one variable. (I used o.)									
65										
66		Left Side	Right Side	Difference						
67	1/f = 1/o + 1/i	0.2	0.2	1.38778E-15	<--- Goal Seek this value to zero by changing o					
68										
69	Now, check the other equations.									
70										
71	foi*(1/f) = foi*(1/o) + 1/I ?	133.333	100.05	33.28333333	<--- far from zero indicates that this equation is false					
72										
73	foi*(1/f) = foi*(1/o + 1/i) ?	133.333	133.3333	9.09495E-13	<--- zero or near zero indicates that this equation is probably true					
74										
75	To test with other values, change f and i, then Goal Seek for the corresponding value of o.									
76										
77	I tested also with these values:									
78		s	p	t						
79		5	6.666667	20						
80		6	7.5	30						
81		3	4.285714	10						

Obviously there is some judgment involved in deciding when a difference is "far from zero". As noted earlier, one good approach is to consider how big the difference is when compared to the size of the biggest number going into the equations. If a difference ever gets as big as one thousandth of the biggest number, you almost certainly have a mistake. Following this rule of thumb, the equation in line 71 is almost certainly wrong, while the one in line 73 is very probably right.

SUMMARY

To recap, the basic principles are simple.

- If two equations are <u>both</u> made <u>true</u> by the <u>same</u> values of their variables, then the equations are probably equivalent. Testing with other values will increase our confidence.

- If one equation is true but the other is clearly not, then the equations are certainly <u>not</u> equivalent, and it's time to track down what went wrong.

- If both equations are false, then you have the wrong values to test with.

If you have not tested <u>both</u> equations with the <u>same</u> values, then you have completely missed the concept, and we need to go over this again.