Algebra Mistakes: How to Make Them, How to Find Them!

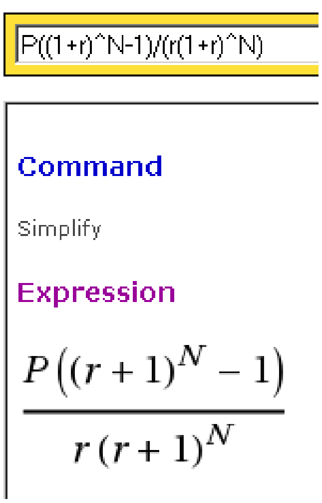
How to Make Them (These are all wrong, of course!)

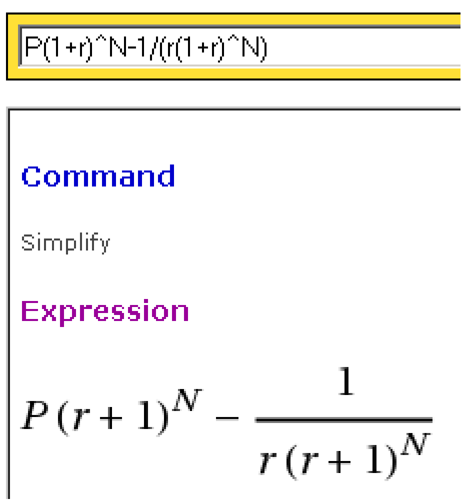
Transcription Errors:	$x - 3 \longrightarrow x \cdot 3, \quad 4 \cdot 3 \longrightarrow 4.3$
Converting from visual notation:	$\frac{a+b}{cd} \longrightarrow (a+b)/cd$
Arithmetic Errors:	$45 + 37 = 72$
Order of operations:	$4 + 3 * 8 \longrightarrow 7 * 8$
Sign Errors:	$5 - (3 - x) \longrightarrow 5 - 3 - x$
Cancellation Errors:	$\frac{ab+c}{b+c} \longrightarrow a$
Expansion Errors:	$5(3x + 2y) \longrightarrow 15x + 2y$
Grouping Errors:	$a = \frac{b}{c+d} \longrightarrow ac + d = b$
Factor Label Errors:	$\frac{5\ miles}{hr} \cdot \frac{15\ minutes}{1} \cdot \frac{60\ minutes}{1\ hr} = x\ miles$
Do Different Things on Each Side:	$V = \frac{1}{3}abc \longrightarrow \frac{1}{3}V = abc$

How to Find Them

Parenthesis errors (converting from visual notation):

- draw brackets & lines connecting matching parentheses;
- convert your single-line expression back to visual notation;
- use Excel 2007's formula debugger, Formulas > Evaluate Formula, to step through execution;
- use QuickMath's Algebra > Simplify to convert your single-line expression back to visual notation (http://www.quickmath.com). Note the difference caused by parentheses here:





Always: go back to the original problem, not just to something you wrote based on the original problem.

Checking arithmetic: do the problem two different ways, for example 1) step by step with a calculator, and 2) convert to single line expression and let Excel or QuickMath do the arithmetic.

Checking a numeric solution to a single equation: plug the number back into the original equation and run through the arithmetic to be sure that you get the same number on both sides.

Checking numeric solutions to simultaneous equations: Plug the numbers back into all of the original equations, and run through the arithmetic to be sure that you get the same number on both sides of each equation.

Story problems: 1) Work the problem backwards. If you computed "drops per minute" from "ml per hour", then take your answer and compute "ml per hour" from it — do you get what you started with? 2) Check to see if your answers make sense. Did you compute that 60 miles per hour = 0.017 feet per second? Think about timing a car as it goes past you. Would it move less than 1 inch in a second?

Checking a symbolic solution to an equation: find numbers that satisfy the original equation and do the arithmetic to be sure those same numbers satisfy the symbolic solution. The same numbers have to work for both the original equation and the solution.

When the original equation is a formula, you can pick any numbers for variables on the right-hand side and just do the arithmetic to find the required variable for the one variable on the left-hand side. If the original equation has complicated expressions on both sides, then it's OK to use Goal Seek to find the value for one variable after picking values for the other variables.

But do **not** use Solver when you're checking equations. The problem is that Solver is very good at finding sets of numbers that work for both equations, even when the equations are wrong! This does not find errors, it hides them!

Consider this example: $r = s/t$, solve for t. Is the answer $t = r/s$ correct? In the first equation $r = s/t$, if you plug in s =100, t =5, then r =20. But then in the second equation r/s=20/100 = 0.2 which is not equal to t since t =5. So no, the equations are not correct, and plugging in the numbers tells you that. But if you apply Solver to this problem, it quickly finds (for example) that r =16.28804, s =16.28804, and t =1 do satisfy both equations. Does this mean that the equations are correct? No! It just means that they are both satisfied by those specific values.[1]

Most pairs of equations have values that make both of them true and will thus hide any errors. It's very unlikely that you would trip over any of those when you're choosing values by hand, but Solver will dig them out like a pig finding truffles. So please, do not use Solver when you're checking.

[1] In fact, both equations will be satisfied by any sets of values where either t = 1or t = -1, and r=s/t.