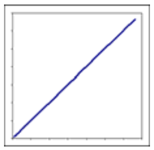
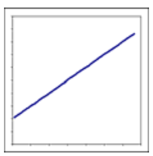
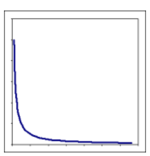
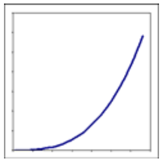
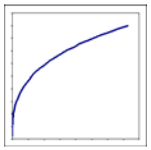
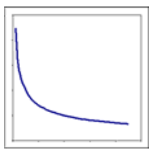
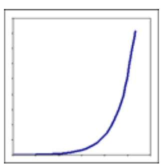
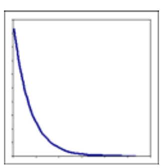
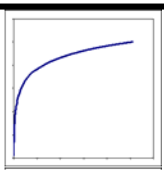
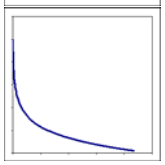
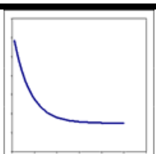
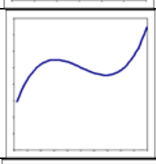
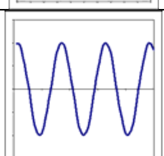
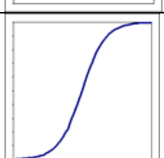
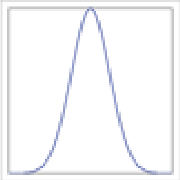
Functional Relationships Commonly Found in Data Graphing, Modeling, and Fitting

Math 101, Littlefield

Thumbnail of graph (using both axes linear)	**Type of Relationship**	**Equation**	**Comments**	**Diagnosed by straight line with these axis types**	**Parameters**	**Formulas to Work Backward from Data**
	Proportional ("varies directly with")	$y = ax$ $(y = ax^1)$	y is a constant multiple of x. Examples: *distance=rate*time* for constant *rate*; *salesTax = 8% * purchasePrice*. Doubling x doubles y.	linear y linear x	a = scaling factor	$a = \frac{y}{x}$
		Excel fits the above with trendline "Linear, set intercept = 0". It will also fit with trendline "Power", exponent = 1.				$x = \frac{y}{a}$
	Linear	$y = ax + b$	Like proportional, but with an offset on one or both axes. Every step in x <u>adds or subtracts</u> some constant <u>increment</u> to y. Linear functions are great for local approximations to other types.	linear y, linear x	a = slope b = y-intercept	$a = \frac{y_2 - y_1}{x_2 - x_1}$
		Excel fits the above with trendline "Linear", no set intercept				$b = y - ax$ $x = \frac{y-b}{a}$
	Reciprocal ("varies inversely with")	$y = \frac{a}{x}$ $(y = ax^{-1})$	This is the partner of a proportional relation, for example $r = d/t$ goes with $d = rt$. "If the trip takes longer, you were going slower." (Larger t, smaller r, constant product.) Doubling x halves y.	logarithmic x logarithmic y (slope = -1)	a = constant product	$a = xy$
		Excel fits the above with trendline "Power", exponent = -1				$x = \frac{a}{y}$

	Power ("varies as the 2^{nd}, 3^{rd}, 4^{th} power of…")	$y = ax^N$	Commonly appears in problems involving area, volume, electrical power, mechanical stress, some chemical equilibrium problems. With $N > 1$, y increases faster with increasing x (concave upward).	logarithmic y logarithmic x (slope = N)	N = power	$N = \dfrac{\ln\left(\frac{y_2}{y_1}\right)}{\ln\left(\frac{x_2}{x_1}\right)}$
	Root ("varies as the 2^{nd}, 3^{rd}, 4^{th} root of…")	$y = ax^N$, with $N = \dfrac{1}{K}$ for K^{th} root	With $0 < N < 1$, y increases more slowly with increasing x (concave downward). Powers and roots occur together, since $y = ax^M$ implies $x = (1/a)\cdot y^{1/M}$. In our pendulum experiment (not done this semester), period $\approx$ length$^{1/2}$			$a = \dfrac{y}{x^N}$
	Reciprocal of power or root	$y = ax^N$, with N < 0	In our pendulum experiment, frequency = 1/period, so frequency $\approx$ 1/length$^{1/2}$ = length$^{-1/2}$			$x = \sqrt[N]{\dfrac{y}{a}} = \left(\dfrac{y}{a}\right)^{\frac{1}{N}}$
		Excel fits the above with trendline "Power", exponent = N				
	Exponential (also called "geometric")	$y = ab^x$	b>1 is exponential growth. Describes unconstrained population growth, compound interest, inflation, number of subsets. Every step in x makes y larger by some constant ratio (multiplication). If maintained, exponential growth rises so quickly that it defies intuition.	logarithmic y linear x (positive slope)	a = amplitude b = growth rate (gives the half life or doubling period)	$b = \left(\dfrac{y_2}{y1}\right)^{\left(\frac{1}{(x_2-x_1)}\right)}$
			b<1 is exponential decay. Describes radioactive and chemical decay (half-life), present value of future $$. Every step in x makes y smaller by some constant ratio (division).	logarithmic y linear x (negative slope)		$a = \dfrac{y}{b^x}$
		Excel fits the above with trendline "Exponential" and a positive or negative coefficient k on the exponent $y = ae^{kx}$, $b = e^k$				$x = \dfrac{\ln\left(\frac{y}{a}\right)}{\ln(b)}$

(same as above)	Exponential (alternate form)	$y = ae^{kx}$	growth(k>0) and decay (k<0), as above for b>1 and b<1	logarithmic y linear x		$k = \dfrac{\ln\left(\dfrac{y_2}{y_1}\right)}{x_2 - x_1}$ $a = \dfrac{y}{e^{kx}}$ $x = \dfrac{\ln\left(\dfrac{y}{a}\right)}{k}$
		Excel fits the above with trendline “Exponential”, $k > 0$ for growth, $k < 0$ for decay.				
	Logarithmic	like exponential growth or decay, but with *x* and *y* interchanged	rises very slowly with increasing *x*.	linear y logarithmic x (positive slope)		
			falls very slowly with increasing *x*.	linear y logarithmic x (negative slope)		
		Excel fits the above with trendline “Logarithmic”				
	Exponential decay to non-zero equilibrium	$y = ab^x + c$		none	a = amplitude b = decay rate c = equilibrium value	numerically fit parameters to data (or measure c directly)
	Polynomial	$y = c_0 + c_1 x + c_2 x^2 + \cdots$	High powers seldom occur naturally, but low powers (2,3,4) can provide very good local approximations to many functions.	none	seldom meaningful	numerically fit parameters to data (obtain from Excel trendline)
	Cyclic	$y = a \cdot \sin(bx + c)$	electronics, cyclic populations	none	a = amplitude b = frequency c = phase	numerically fit parameters to data
	Logistic Equation	$y = \dfrac{1}{1 + \left(\dfrac{1}{a} - 1\right)e^{-bx+c}}$	population growth with resource constraints	none		numerically fit parameters to data

	Bell Curve	$y = ae^{-\left(\frac{x-b}{c}\right)^2}$	This appears as a probability density function for the “normal distribution”. It is also a useful function to describe anything that goes up and then comes back down symmetrically, especially if you have reason to believe that the tails are exponential.	none	a = peak height b = peak location c = width	numerically fit parameters to data