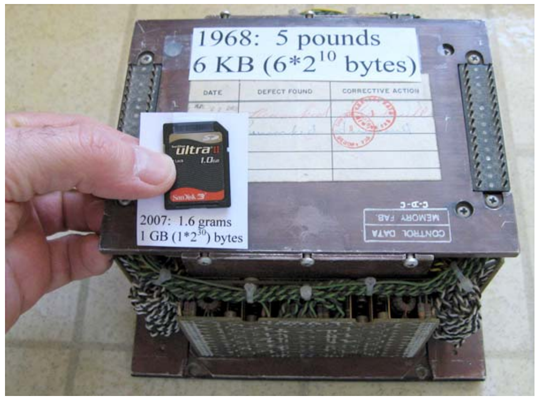
Exponential Improvement of Memory Density

Math 101, Littlefield



The above picture shows representative units of computer memory from 1968 and 2007.

In 1968, magnetic core technology yielded a memory module of 6 KB (yes, 6 thousand bytes) using 5 pounds of material.

In 2007, compact flash silicon technology yielded a memory module of 1 GB (one billion bytes) using 1.6 grams of material.

In terms of memory density (bytes per gram), this represents an improvement of a factor of roughly 2.5×10^8 (250 million).[1]

Assuming that the improvement was exponential, what was the rate?

There are a couple of good ways to think about this. One of them is to ask about the growth factor per year, a second is to ask about the doubling period.

To find the growth factor per year, we first compute the number of years (2007-1968=39) and then ask what value of g is required such that $g^{39} = 2.5 \times 10^8$. We can do that with powers or logarithms[2], or Goal Seek or Solver. The answer is $\boxed{g = 1.64 \text{ (64\% per year)}}$.

To find the doubling period, we might first ask how many doubling periods are required. That is, find N such that $2^N = 2.5 \times 10^8$. Again, we can do that with logarithms[3] or with Goal Seek or Solver. The answer is $N = 27.9$. Then simply divide 39 years by 27.9 periods to find the $\boxed{\text{doubling period } P = 1.40 \text{ years}}$, or in other units, $\boxed{P = 16.8 \text{ months}}$.

When people speak of Moore's Law – "doubling every 18 months" – this is what they're talking about.

By the way, you might notice that this growth rate is so rapid that Dr.Bartlett's "Rule of 70" starts to lose accuracy. At 64% per year, the Rule of 70 would predict a doubling period of 70/64 = 1.09 years, not the actual 1.40 years.

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[1] The calculation is (1GB / 1.6gm) / (6KB / (5 lb * 454 gm/lb)).

[2] Using powers, $g = (2.5 \times 10^8)$^(1/39). Using logarithms, $\log_{10}(g^{39}) = \log_{10}(2.5 \times 10^8)$, so $39 * \log_{10}(g) = \log_{10}(2.5 \times 10^8)$, so $\log_{10}(g) = 8.398/39 = 0.215$, $g = 10^{0.215} = 1.66$. Goal Seek or Solver would be much more reliable!

[3] The calculation is $\log_{10}(2^N) = \log_{10}(2.5 \times 10^8)$, so $N * \log_{10}(2) = 8.398$, N = 8.398 / $\log_{10}(2)$ = 28.4 .