Cancelling in Algebraic Fractions

An "algebraic fraction", for our purposes, is simply any fraction whose numerator and denominator contain variables instead of just integers.

Often we are interested in fractions whose numerators and denominators involve only addition, subtraction, and multiplication (including powers). Examples include $\frac{a}{b}$, $\frac{a+bc}{def-g}$, and $\frac{x^4}{x^2}$.

For fractions like these, it makes sense to "cancel common factors" or "reduce to lowest terms".

Those phrases mean essentially the same thing with algebraic fractions that they mean with ordinary number fractions: find something that divides evenly into both the numerator and denominator, and get rid of it.

Here are some examples:

$$\frac{x^5}{x^2} = \frac{(x^2)\cdot(x^3)}{(x^2)\cdot(1)} = \frac{x^3}{1} = x^3 \qquad \text{cancel the common factor } x^2$$

$$\frac{(a+b)x}{(a+b)} = \frac{(a+b)\cdot(x)}{(a+b)\cdot(1)} = \frac{x}{1} = x \qquad \text{cancel the common factor } (a+b)$$

$$\frac{ac+bc}{cd} = \frac{(a+b)c}{cd} = \frac{(c)\cdot(a+b)}{(c)\cdot(d)} = \frac{(a+b)}{d} = \frac{a+b}{d} \qquad \text{cancel the common factor } c$$

Notice the pattern: you can cancel a factor (f) if ***and only if*** you can transform the fraction into this exact form: $\frac{(f)\cdot(a)}{(f)\cdot(b)}$

If you cannot put the fraction into that form — typically because there are additions or subtractions that get in the way — then you can <u>not</u> cancel anything because there are no common factors!

Examples where you can <u>not</u> cancel are:

$$\frac{fa+d}{fb} \text{ and } \frac{fa-d}{fb} \qquad \text{addition or subtraction in the numerator}$$

$$\frac{fa}{fb-d} \text{ and } \frac{fa}{fb-d} \qquad \text{addition or subtraction in the denominator}$$

In general, if you are not sure that you can cancel something in a fraction, then don't even try.

It is far better to end up with a fraction that is <u>correct</u> but contains common factors, than it is to end up with a fraction that is <u>wrong</u> because it has been canceled incorrectly.

When in doubt, check your work by plugging in numbers. Do the arithmetic of the original expression and of the canceled one, and make sure that they both produce the same value!

The following is a very common MISTAKE:

$$\frac{fa+d}{fb} = \frac{\cancel{f}a+d}{\cancel{f}b} = \frac{a+d}{b}$$

Why is this wrong?

Well, let's suppose that it's correct, and see where that takes us.

$\frac{fa+d}{fb} = \frac{a+d}{b}$	what we're supposing
$\frac{fa+d}{fb} = \frac{f}{f} \cdot \frac{a+d}{b}$	multiply right-hand side by 1, in the form of $\frac{f}{f}$
$\frac{fa+d}{fb} = \frac{f(a+d)}{fb}$	reformat the right-hand side so that the denominator looks like the left-hand side
$fa + d = f(a + d)$	numerators must be equal, because the fractions are equal and the denominators are equal
$fa + d = fa + fd$	distribute the multiplication on the right-hand side
$d = fd$	subtract fa from both sides
$1d = fd$	just being very explicit here...
$d = 0$ $f = 1$	this is one way that we can have $1d = fd$ and this is the other

In other words, the only way that it's possible to have $\frac{fa+d}{fb} = \frac{a+d}{b}$ is if either $d = 0$ or $f = 1$.

But then either the addition or the multiplication wasn't actually doing anything!

So, the reason that canceling in this case is wrong is that it gives an equation that is untrue for almost all numbers.

There are similar analyses for all other cases involving plus + or minus – appearing outside parentheses on either top or bottom of a fraction.

You cannot cancel while you have a <u>plus or minus outside parentheses</u> on either the top or the bottom of a fraction.

If you can get plus and minus wrapped inside parentheses without changing the values, then you can think about canceling the whole parenthesis group.