Math 101, Littlefield[1]
Algebra — The Numbers Game

Here is a trick that may help you in learning to solve algebra problems.

Suppose, for example, you have this problem: Given $A = \frac{1}{2}h(b + B)$, solve for *B*.

Step 1. Make a small table listing all your variables

Next to each variable except the one you're solving for, write a different 3-digit number. Here are some excellent big ugly 3-digit numbers to choose from:
127, 163, 223, 251, 307, 349, 419, 457, 541, 599, 643, 691, 751, 797, 839, 887, 937, 983.

For example:

Variable	Big ugly 3-digit number
A	349
h	163
B	*none — we're solving for B*
b	751

Step 2. Rewrite your original equation, substituting the selected 3-digit numbers in place of every variable except the one that you're solving for. The resulting equation will contain nothing except the variable you care about, some big ugly 3-digit numbers, and some arithmetic operators (+ – * /) etc. Here is what you get in the current example:

$$349 = \frac{1}{2} * 163 * (751 + B)$$

Step 3. Go through the motions of solving this new equation, but do not actually carry out any arithmetic on the big ugly 3-digit numbers. Just indicate what arithmetic is required.

In fact, with the big ugly numbers, you are permitted to use only the basic identities: u-u=0, u+0=u, u/u=1, 1*u=u, $(a+b)u=au+bu$, and so on. You are also permitted to do arithmetic on any numbers that originally appeared in the equation, or that appear as the solution evolves. For example, you are permitted to substitute $u+2u \rightarrow 1u+2u \rightarrow (1+2)u \rightarrow 3u$.

You will end up with an equation still full of big ugly 3-digit numbers.
(See table on next page for the solution steps.)

$$B = \frac{2 * 349 - 163 * 751}{163}$$

Step 4. Go through the solved equation, replacing every big ugly 3-digit number with its equivalent variable name. Voila — you now have a symbolic solution!

$$B = \frac{2A - hb}{h}$$

Example: From the homework...

Solve $A = \frac{1}{2}h(b + B)$ for *B*.

Here is the table of substitutions.

Variable	3-digit number
A	349
h	163
b	751

And here are the solution steps.

Equation	Explanation
$A = \frac{1}{2}h(b + B)$	equation as given
$349 = \frac{1}{2} * 163 * (751 + B)$	replace variables by 3-digit numbers
$2 * 349 = 2 * \frac{1}{2} * 163 * (751 + B)$	multiply both sides by 2 to "kill the denominators"
$2 * 349 = 163 * (751 + B)$	note that 2*(1/2) = 1, and simplify
$2 * 349 = 163 * 751 + 163 * B$	expand to "get rid of the parentheses"
$2 * 349 - 163 * 751 = 163 * B$	isolate your variable terms on one side
$2 * 349 - 163 * 751 = (163) * B$	find the coefficient on the variable
$\frac{2 * 349 - 163 * 751}{(163)} = \frac{(163) * B}{(163)}$	divide both entire sides by that coefficient. Notice that we're being very careful to divide the <u>entire</u> left hand side by the coefficient.
$\frac{2 * 349 - 163 * 751}{163} = B$	simplify right side, remove unnecessary parentheses
$B = \frac{2 * 349 - 163 * 751}{163}$	put variable on the left, just because it's traditional
$B = \frac{2A - hb}{h}$	replace 3-digit numbers with equivalent variable names, and eliminate unnecessary asterisks

These solution steps are all based on our standard strategy:

1. **Kill the denominators!** *<u>Multiply</u> both sides by whatever is in the denominators.*
2. **Expand to eliminate the parentheses.** *<u>Distribute</u> multiplication across addition & subtraction. Fully expand both sides so that you have at worst a sum of products.*
3. **Isolate your variable terms on one side.** *<u>Add and subtract</u> entire terms.*
4. **Find the coefficient on the variable.** *<u>Factor</u> into* (coefficient)*variable
5. **<u>Divide</u> both sides by the coefficient.**
6. **<u>Simplify</u> the result (if desired!)**