Math 101: Using Tables to Solve Mixture Problems

Mixture problems come up fairly frequently in the real world. Here are a couple of examples:

a. Joe Grocer wants to mix 50 pounds of nuts that cost $2 per pound, with some other nuts that cost $6 per pound, to make a mixture that costs $5 per pound. How many pounds of $6 nuts should he use?

b. Beth is planning to invest $18,000, putting some in a relatively safe account that's expected to return a gain of 2.25% per year, and the rest in a riskier account that's expected to return 6.5%. How much money should she put in each account to get an expected gain of $660 over the next year? [1]

Generally the best way to solve a mixture problem is to make a table. The table contains one row for each type of "item" that's being mixed, and one column for every fact you know about that item.

When you're done, the table will have some holes in it, corresponding to information you don't have. To solve the problem, you fill in the holes, by working from whatever other information you do have.

For the mixed-nuts problem, a typical table would start out like this:

Item	**Number of pounds**	**Dollars per pound**	**Dollars for the item**
Type A nuts	50	2	
Type B nuts		6	
mixture		5	

Now, what are we asked to find?? The number of pounds of Type B nuts – the ones at $6 per pound. We're going to be doing some algebra, so stick an x in that cell.

Item	**Number of pounds**	**Dollars per pound**	**Dollars for the item**
Type A nuts	50	2	
Type B nuts	x	6	
mixture		5	

Now fill in other cells, using our general knowledge about how prices and pounds multiply and add together.

Item	**Number of pounds**	**Dollars per pound**	**Dollars for the item**
Type A nuts	50	2	50 * 2
Type B nuts	x	6	x * 6
mixture	50 + x	5	

I've left that bottom right cell blank for a moment, because I want to call attention to it. The bottom right cell is special in these tables, because there are two different ways to express the number that has to end up there.

[1] The investment problem is a paraphrasing of problem 4 on page 352 of your textbook, volume I.

One way is to add down the last column: dollars for the mixture equals dollars for Type A nuts plus dollars for Type B nuts. As math: DollarsForMixture = 50*2 + *x**6 .

Another way is to multiply across the last row: dollars for the mixture equals number of pounds of mixture, times dollars per pound for the mixture. As math: DollarsForMixture = (50+*x*)*5 .

Let's put those expressions in our table so we can see them more easily.

Item	**Number of pounds**	**Dollars per pound**	**Dollars**
Type A nuts	50	2	50*2
Type B nuts	*x*	6	*x* * 6
mixture	50 + *x*	5	50*2 + *x* * *6* (adding down) = (50 + *x*)*5 (multiplying across)

Notice the "=" sign in that bottom right cell. It's there because DollarsForMixture is only one number. Both ways of computing that number have to produce the same number. That's the key fact that lets us solve this problem, because it lets us write this equation:

$$50 * 2 + x * 6 = (50 + x) * 5$$

If you look at just the equation, you'll find it's very hard to make sense of! The trick is to look back at the table, so you can see where each of the parts comes from.

In any case, now that we have this equation, it's easy to solve it:

$$50 * 2 + x * 6 = (50 + x) * 5$$
$$100 + x * 6 = 250 + x * 5$$
$$x * 6 - x * 5 = 250 - 100$$
$$x = 150$$

So there's our answer: Joe Grocer needs 150 pounds of the $6 per pound nuts, the ones that we're calling Type B.

But we're not done — we need to check our answer. Plug it back in and see if it works.

50 pounds at $2 per pound gives $100 for Type A nuts.
150 pounds at $6 per pound gives $900 for Type B nuts.
50 pounds of Type A nuts plus 150 pounds of Type B nuts gives 200 pounds of mixture
$100 for Type A nuts plus $900 for Type B nuts gives $1000 for the mixture.
$1000 for 200 pounds of mixture gives a cost of $5 per pound for the mixture, which is what the problem said that Joe needed to make.

Excellent: the solution checks!

Now let's do the other one. It's a little harder because we're asked to find <u>two</u> unknowns, the amount to be placed in each type of account. For convenience, let's repeat the problem:

> *Beth is planning to invest $18,000, putting some in a relatively safe account that's expected to return a gain of 2.25% per year, and the rest in a riskier account that's expected to return 6.5%. How much money should she put in each account to get an expected gain of $660 over the next year?*

First, let's just fill in what we know:

Item	**Dollars invested**	**Expected gain (as a rate)**	**Dollars gained**
safe investment		2.25% = 0.0225	
risky investment		6.5% = 0.065	
mixed investment	$18,000		$660

Now, what are we asked to find?? We need to fill in those blank cells in the upper left, the dollars invested in each category. We could use two variables (x and y), but since we haven't studied how to solve for two variables, let's work with just one. Stick an x in one of the empty cells.

Item	**Dollars invested**	**Expected gain (as a rate)**	**Dollars gained**
safe investment	x	2.25% = 0.0225	
risky investment		6.5% = 0.065	
mixed investment	$18,000		$660

Now fill in the rest of the cells, using our general knowledge about how rates work and how dollars add up.

Item	**Dollars invested**	**Expected gain (as a rate)**	**Dollars gained**
safe investment	x	2.25% = 0.0225	0.0225 * x
risky investment	18,000 - x	6.5% = 0.065	0.065 * (18,000 – x)
mixed investment	$18,000	$660 / $18,000 = 3.6667%	0.0225 * x + 0.065 * (18,000 – x) = $660

Again, that bottom right cell can be written two ways. One way is the $660 that was given in the problem. The other way is that algebra expression that we developed by filling out the table. Their values have to be equal, so we end up with an equation to solve:

$$0.0225 * x + 0.065 * (18{,}000 - x) = 660$$

From here, it's pretty straightforward:

$$0.0225 * x + 0.065 * (18{,}000 - x) = 660$$
$$0.0225 * x + 0.065 * 18{,}000 - 0.065 * x = 660$$
$$0.0225 * x - 0.065 * x = 660 - 0.065 * 18{,}000$$
$$(0.0225 - 0.065) * x = 660 - 1170$$
$$(-0.0425) * x = -510$$
$$x = \frac{-510}{-0.0425} = 12{,}000$$
$$18{,}000 - x = 6{,}000$$

And there's our answer: $12,000 in the safe investment at 2.25%, and $6,000 in the risky investment at 6.5%.

Does the answer check?

$12,000 at 2.25% gives a gain of 12,000 * 0.0225 = $270 from the safe investment.
$6,000 at 6.5% gives a gain of 6,000 * 0.065 = $390 from the risky investment.
$270 + $390 = $660, the total return that the problem said Beth needed.

Yep, it checks!

Summary of the method:

1. **Build a table.** Allocate one row for each type of item, and one column for each fact you know about that item.
2. **Fill in values that you know**.
3. **Stick an *x* in one cell that you need to figure out**.
4. **Fill in the other cells based on what you know about how quantities multiply and add**.
5. **If there are two different ways to know the value of a cell, use those to write an equation**.
6. **Solve the equation**.
7. **Write your answer in terms of the original problem**.
8. **Check your answer** — plug it back in to see if it works!